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Aerodynamics of Wings in Subsonic Shear Flow

A. Barsony-Nagy* and M. Hanin†
Technion—Israel Institute of Technology, Haifa, Israel

Lifting-surface and lifting-line theories are developed for wings in a nonuniform subsonic parallel stream whose velocity and density vary in the vertical direction. A lifting-surface integral equation is derived from linearized compressible Euler equations by using a Fourier transformation method. A lifting-line integral equation for a nonuniform subsonic stream is then deduced and a method for solving this equation is outlined. Numerical results are given for the lift and induced drag of elliptic wings in a jet stream, a wake stream, and a monotonic sheared stream.

Introduction

THE purpose of this paper is to develop a method for calculating the flow and the aerodynamic forces on a three-dimensional wing in a subsonic nonuniform parallel stream whose velocity and density vary in the vertical direction. This problem has a practical significance since it relates to flight in a wake, a jet, or a sheared wind. Effects of a nonuniform stream can be appreciable also for a wing located behind a canard surface, for a tail situated in the wake of a wing, and for a wing or tail situated in propeller slipstream. From a more theoretical viewpoint, it is of interest to find out how the flow and the aerodynamic forces behave when the usual assumption of a uniform stream is relinquished, so that the velocity and density of the unperturbed stream are allowed to have vertical gradients.

The literature on the subject is not very extensive, owing probably to the difficulties in treating rotational flow. von Kármán and Tsien[1] considered a lifting-line theory for an incompressible nonuniform stream and outlined a method for determining the induced drag for a given lift distribution. Dowell,[2] Ventres,[3] and Williams et al.[4] obtained numerical lifting-surface solutions for panels and wings in a shear layer with a power law velocity profile simulating a turbulent boundary layer. Homentcovschi and Barsony-Nagy[5] formulated a lifting-surface integral equation for nonuniform incompressible flow with a general stream velocity profile. Hanin and Barsony-Nagy[6] developed a slender wing theory for a subsonic or low-supersonic nonuniform stream.

In the present paper, lifting-surface and lifting-line equations are obtained for wings in a subsonic nonuniform stream. It is assumed that the perturbations due to the wing are small, but no simplifying assumption is made with regard to the velocity and density profiles of the stream, except for the requirements that the Mach number profile $M(z)$ has a nonzero value $M(0)$ and a finite second-order derivative $M''(0)$ at the wing plane, and that $|M'(z)|$ is integrable across the stream. Solutions of the lifting-line equation are calculated for elliptic wings in subsonic streams with several Mach-number profiles. A brief parametric study is made of the effects of a nonuniform stream on lift and induced drag.

Lifting-Surface Theory

We consider first the general case of a wing with arbitrary planform and aspect ratio. The wing is set at a small angle of attack in a nonuniform stream parallel to the x axis. The y axis is taken in the spanwise direction and the wing is situated near the $z=0$ plane. The undisturbed velocity $U(z)$, density $\rho(z)$, and Mach number $M(z)$ of the stream are given functions of the vertical coordinate z. The undisturbed pressure does not vary with z since the stream is parallel.

The flow due to the wing is regarded as a small perturbation. The equations of flow, consisting of the Euler equations of momentum, continuity, energy, and state for an inviscid perfect gas, are linearized for small deviations from the nonuniform stream. The set of linearized equations yields a single differential equation for the pressure perturbation p

$$[1-M^2(z)]p_{xx}+p_{yy}+p_{zz}-\frac{2M'(z)}{M(z)}p_z=0 \tag{1}$$

where the subscript notation for the partial derivatives is used. In a different context, Eq. (1) was given first by Lighthill.[7]

For a wing with zero thickness, the linearized boundary condition on its surface is

$$w(x,y,0)=-U(0)\alpha(x,y) \tag{2}$$

where $\alpha(x,y)$ is the local angle of attack and w the upward velocity component. To express this condition in terms of the pressure perturbation p we use the linearized momentum equation in the z direction

$$\rho(z)U(z)w_x+p_z=0 \tag{3}$$

The boundary condition [Eq. (2)] then becomes

$$\int_{-\infty}^{x} p_z(\xi,y,0)\,\mathrm{d}\xi=\rho(0)U^2(0)\alpha(x,y) \tag{4}$$

The pressure perturbation is discontinuous when crossing the wing surface, and we have

$$p(x,y,-0)-p(x,y,+0)=\ell(x,y) \tag{5}$$

where $\ell(x,y)$ is the pressure load. From Eqs. (2) and (3) it follows that the derivative p_z is continuous for a wing without thickness, so that

$$p_z(x,y,-0)-p_z(x,y,+0)=0 \tag{6}$$

An integral equation for the pressure load $\ell(x,y)$ will be obtained by solving the differential equation (1) with the jump conditions [Eqs. (5) and (6)], and then applying the boundary condition [Eq. (4)].

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*Lecturer, Dept. of Aeronautical Engineering. Member AIAA.
†Professor, Dept. of Aeronautical Engineering. Member AIAA.

To solve Eq. (1), a Fourier transformation is made with respect to x and y. Any function $f(x,y,z)$ is related to its transform $\tilde{f}(k_1,k_2,z)$ by

$$f(x,y,z) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\tilde{f}(k_1,k_2,z)e^{i(xk_1+yk_2)}\mathrm{d}k_1\mathrm{d}k_2 \qquad (7)$$

Equations (1), (5), and (6) then give

$$\tilde{p}_{zz} - \frac{2M'(z)}{M(z)}\tilde{p}_z - [(1-M^2(z))k_1^2+k_2^2]\tilde{p}=0 \qquad (8)$$

$$\tilde{p}(k_1,k_2,+0)-\tilde{p}(k_1,k_2,-0) = -\tilde{\ell}(k_1,k_2) \qquad (9)$$

$$\tilde{p}_z(k_1,k_2,+0)-\tilde{p}_z(k_1,k_2,-0)=0 \qquad (10)$$

For a subsonic stream the perturbations vanish at large distances above and below the wing, and we have

$$\tilde{p}(k_1,k_2,\pm\infty)=0 \qquad (11)$$

The transform solution $\tilde{p}$ is the product of the transformed load $\tilde{\ell}$ and a fundamental solution $\tilde{P}$

$$\tilde{p}(k_1,k_2,z) = -\tilde{\ell}(k_1,k_2)\tilde{P}(k_1,k_2,z) \qquad (12)$$

where $\tilde{P}$ is the solution of the ordinary differential equation (8) satisfying the conditions

$$\tilde{P}(k_1,k_2,+0)-\tilde{P}(k_1,k_2,-0)=1 \qquad (13a)$$

$$\tilde{P}_z(k_1,k_2,+0)-\tilde{P}_z(k_1,k_2,-0)=0 \qquad (13b)$$

$$\tilde{P}(k_1,k_2,\pm\infty)=0 \qquad (13c)$$

The function $\tilde{P}$ and its inverse $P(x,y,z)$ depend only on the Mach-number profile of the unperturbed stream and are independent of the wing geometry or angle of attack. They can be regarded as the extension into the subsonic range of the incompressible fundamental solution studied by Lighthill.[8] The existence of the subsonic fundamental solution was investigated by Barsony-Nagy,[9] who found that a sufficient condition for the existence of $\tilde{P}(k_1,k_2,z)$ is the integrability of $|M'(z)|$ in the range $-\infty<z<\infty$.

When the function $\tilde{P}$ and its inverse $P(x,y,z)$ have been determined, the pressure perturbation p is expressed in terms of the load $\ell(x,y)$ by applying the convolution theorem to Eq. (12), which gives

$$p(x,y,z) = -\frac{1}{2\pi}\iint_S \ell(\xi,\eta)P(x-\xi,y-\eta,z)\mathrm{d}\xi\mathrm{d}\eta \qquad (14)$$

where S denotes the wing area. The boundary condition [Eq. (4)] now yields an integral equation for the pressure load $\ell(x,y)$ on the wing,

$$\iint_S \ell(\xi,\eta)I(x-\xi,y-\eta)\mathrm{d}\xi\mathrm{d}\eta = -2\pi\rho(0)U^2(0)\alpha(x,y) \qquad (15)$$

where

$$I(x-\xi,y-\eta) = \int_{-\infty}^{x-\xi} P_z(\xi,y-\eta,0)\mathrm{d}\xi \qquad (16)$$

The kernel I is singular. An expression exhibiting its singularities is obtained by expanding asymptotically the fundamental transform solution $\tilde{P}_z$ for large values of $k_1^2+k_2^2$, inverting the terms of the expansion, and integrating as indicated in Eq. (16). This gives

$$I(x-\xi,y-\eta) = \frac{1}{2}\Big[\frac{1}{(y-\eta)^2}\Big(1-\frac{x-\xi}{r}\Big) - \sigma\ln\Big(\frac{\sqrt{|\sigma|}}{\beta}(x-\xi+r)\Big) + N(x-\xi,y-\eta)\Big] \qquad (17)$$

where $N(x-\xi,y-\eta)$ is a bounded function depending only on the Mach number profile of the stream, and we denote for brevity

$$\beta=\sqrt{1-M^2(0)} \qquad (18)$$

$$\sigma=M''(0)/2M(0) \qquad (19)$$

$$r=\sqrt{(x-\xi)^2+\beta^2(y-\eta)^2} \qquad (20)$$

The function $N(x-\xi,y-\eta)$ can be evaluated by solving the differential equation (8) with the boundary conditions [Eqs. (13)] for $\tilde{P}$, inverting by means of Eq. (7) near $z=0$, calculating I from Eq. (16), and subtracting the singular terms as indicated in Eq. (17). In a numerical computation the transforms of the singular terms should be subtracted before inverting, since then the singularities would be avoided.

The lifting-surface integral equation resulting from Eqs. (15) and (17) is

$$\iint_S \ell(\xi,\eta)\Big[\frac{1}{(y-\eta)^2}\Big(1+\frac{x-\xi}{r}\Big) - \sigma\ln\Big(\frac{\sqrt{|\sigma|}}{\beta}(x-\xi+r)\Big) + N(x-\xi,y-\eta)\Big]\mathrm{d}\xi\mathrm{d}\eta = -4\pi\rho(0)U^2(0)\alpha(x,y) \qquad (21)$$

Here the first term coincides with the subsonic lifting-surface equation for uniform stream. The second and third terms are due to the stream nonuniformity. We note that the logarithmic term involves the second derivative of the Mach number profile at the wing plane $M''(0)$. It may be expected that this derivative will have a marked effect on the resulting pressure load and lift forces. A nondimensional parameter characterizing this effect is $M''(0)s^2/M(0)$, where s denotes the semispan.

Equation (21) can be regarded as an extension to the subsonic range of the incompressible lifting-surface equation given by Homentcovschi and Barsony-Nagy.[5] In the case of small aspect ratio, Eq. (21) approaches the slender wing equation for nonuniform stream obtained by the present authors.[6] On the other hand, Eq. (21) differs essentially from the integral equations given by Ventres[3] and Williams et al.[4] for wings in a sheared stream simulating a turbulent boundary layer. The differences are due to the special form of the boundary conditions assumed by Ventres and Williams et al. to account for the vanishing of velocity on the wing surface in their case.

To calculate solutions of the lifting-surface equation (21), it would be necessary to compute first the term $N(x-\xi,y-\eta)$ of the kernel, which would require a time-consuming numerical inversion of two-dimensional Fourier transforms. In what follows, we consider the case of a wing with high aspect ratio, for which the integral equation (21) can be reduced to a lifting-line equation. In this case the Fourier transforms are one-dimensional and can be inverted readily.

Lifting-Line Approximation

The wing is now assumed to be unswept or slightly swept and to have a large aspect ratio. Denoting by $2s$ the span, we have on the wing

$$\epsilon = \max|x-\xi|/2s \ll 1 \qquad (22)$$

The lifting-surface equation (21) is approximated for small ϵ by expanding the kernel and retaining terms of the lowest order in ϵ. The following equation is obtained

$$\int_{-s}^{s} L(\eta)\left[\frac{1}{(y-\eta)^2}-\sigma \ln|\sigma^{1/2}(y-\eta)|+N(y-\eta)\right]d\eta$$

$$-2\beta\int_{x_\ell(y)}^{x_t(y)}\frac{\ell(\xi,y)}{x-\xi}d\xi=-4\pi\rho(0)U^2(0)\alpha(x,y) \quad (23)$$

Here $L(y)$ is the spanwise lift distribution

$$L(y)=\int_{x_\ell(y)}^{x_t(y)}\ell(\xi,y)d\xi \quad (24)$$

$x_\ell(y)$ and $x_t(y)$ are the x coordinates of the leading and trailing edges, and we denote

$$N(y-\eta)=N(0,y-\eta) \quad (25)$$

An integral equation for the lift distribution $L(y)$ is obtained by multiplying Eq. (23) by the weight factor $\{[x-x_\ell(y)]/[x_t(y)-x]\}^{1/2}$ and integrating with respect to x over the chord. The resulting lifting-line equation is

$$\beta L(y)=\pi\rho(0)U^2(0)\alpha(y)+\frac{1}{2}c(y)\int_{-s}^{s}L(\eta)\left[\frac{1}{(y-\eta)^2}\right.$$

$$\left.-\sigma\ln|\sigma^{1/2}(y-\eta)|+N(y-\eta)\right]d\eta \quad (26)$$

where $c(y)$ is the local chord and $\alpha(y)$ is the weighted average angle of attack, defined by

$$\alpha(y)\doteq\frac{2}{c(y)}\int_{x_\ell(y)}^{x_t(y)}\sqrt{\frac{x-x_\ell(y)}{x_t(y)-x}}\,\alpha(x,y)dx \quad (27)$$

In the integral equation (26), the last two terms are due to the stream nonuniformity and they depend on the Mach-number profile only. The preceding terms coincide with Prandtl's lifting-line equation for uniform stream.

We note that the subsonic lift distribution $L(y)$ obeys a similarity rule which relates it to an incompressible flow. In fact, Eq. (26) shows that the lift distribution $L(y)$ is the same as that on a wing having the stretched chord $c(y)/\beta$ set at the same angle of attack in an incompressible sheared stream whose velocity profile and density are $U(0)M(z)/M(0)$ and $\rho(0)$, respectively.

To solve the integral equation (26), we use a standard Fourier series expansion along the span. The coordinates are changed by setting

$$y=s\cos\phi,\quad \eta=s\cos\theta \quad (28)$$

and the lift distribution is expanded in the form

$$L(y)=\rho(0)U^2(0)s\sum_{m=1}^{\infty}L_m\sin(m\phi) \quad (29)$$

The integral equation (26) yields now an infinite set of algebraic equations for the coefficients L_m of the series [Eq. (29)]. The logarithmic term of the integral equation can be treated in the same way as in Ref. 6. In solving the infinite set approximately we take a finite number of terms and equations. In most cases, six terms of the series give an adequate accuracy.

The lift L of the wing is related to the first coefficient L_1 only

$$L=\int_{-s}^{s}L(y)dy=\frac{\pi}{2}\rho(0)U^2(0)s^2L_1 \quad (30)$$

Induced Drag and Pitching Moment

To calculate the induced drag of a high-aspect-ratio wing in nonuniform stream, we may use the classical concept of downwash or induced angle of attack. From Weissinger's theory of two-dimensional airfoils[10] we know that the D'Alembert paradox holds in a nonuniform stream. It follows that the induced drag is due to the deflection of the lift by the downwash, so that

$$D_i=\int_{-s}^{s}\alpha_i(y)L(y)dy \quad (31)$$

where D_i denotes the induced drag and α_i is the local downwash angle. To find α_i we refer to the integral equation of two-dimensional airfoil in nonuniform stream given by Weissinger[10]

$$\int_{x_\ell}^{x_t}\ell(\xi)\left[\frac{1}{x-\xi}-Q(x-\xi)\right]d\xi=2\pi\beta^{-1}\rho(0)U^2(0)\alpha(x) \quad (32)$$

Here Q is a continuous odd function which depends on the Mach number profile only. Its order of magnitude is $Q=0((x-\xi)/h^2)=0(\epsilon s/h^2)$, where h denotes a length characterizing the vertical extent of the stream nonuniformity. The drag calculation is simplified by assuming $h/s=0(1)$ or $\gg 1$. Then the contribution of the term Q in Eq. (32) has the relative magnitude $0(\epsilon^2)$ and can be neglected. For the three-dimensional wing we must include in Eq. (32) the induced downwash α_i, which gives

$$\int_{x_\ell(y)}^{x_t(y)}\frac{\ell(\xi,\eta)}{x-\xi}d\xi=2\pi\beta^{-1}\rho(0)U^2(0)[\alpha(x,y)-\alpha_i(y)] \quad (33)$$

The downwash angle is found by comparing Eq. (33) with the lifting-line equation (23), which shows that

$$\alpha_i(y)=-\frac{1}{4\pi\rho(0)U^2(0)}\int_{-s}^{s}L(\eta)\left[\frac{1}{(y-\eta)^2}\right.$$

$$\left.-\sigma\ln|\sigma^{1/2}(y-\eta)|+N(y-\eta)\right]d\eta \quad (34)$$

The induced drag is now obtained by introducing the downwash angle α_i from Eq. (34) into Eq. (31).

To find the pitching moment of high-aspect-ratio wings, we observe that when the function $\alpha_i(y)$ has been determined, Eq. (33) has the same form as the two-dimensional thin-airfoil equation for a uniform stream. Thus, the pressure load is affected by the stream nonuniformity through the sectional downwash angle α_i only. It follows that the location of the aerodynamic center of each section is the same as in a uniform stream, and that the sectional pitching moment about the aerodynamic center does not depend on the stream nonuniformity.

Examples and Numerical Results

To examine the effects of a nonuniform stream on the lift and induced drag, solutions of the lifting-line equation were calculated for a plane elliptic wing and several Mach number profiles, representing flight in a jet, in a wake, in a linearly sheared wind, and in a nonlinear sheared wind.

The jet stream and wake stream profiles are described by

$$M(z)=M_1+(M_0-M_1)e^{-z^2/h^2} \quad (35)$$

Here the wing location $z=0$ is taken at the symmetry plane of the stream, and $M_0=M(0)$ is the local Mach number there. The external Mach number is M_1 and the length h is related to the vertical extent of the jet or wake.

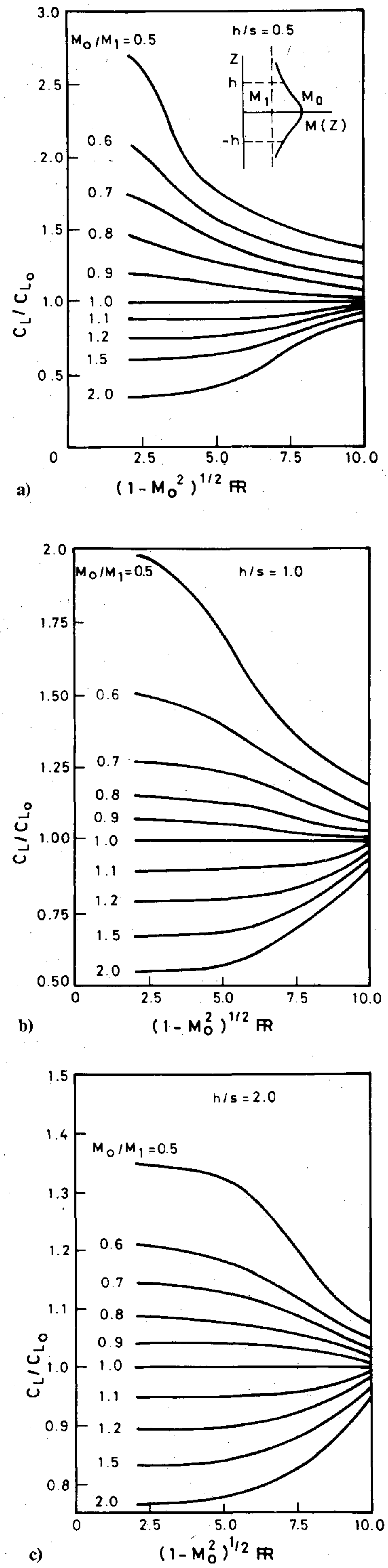

Fig. 1 Lift coefficient in jet stream and wake stream.

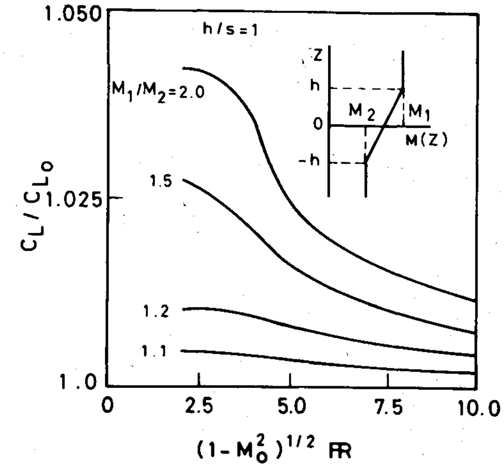

Fig. 2 Lift coefficient for linear Mach number profile.

The linear shear profile, with the wing placed at the middle plane, is given by

$$M = M_1 \qquad \text{for } z \geq h$$
$$= \tfrac{1}{2}(M_1 + M_2) + \tfrac{1}{2}(M_1 - M_2)\frac{z}{h} \qquad \text{for } -h \leq z \leq h$$
$$= M_2 \qquad \text{for } z \leq -h \qquad (36)$$

A nonlinear monotonic profile was chosen as

$$M(z) = \tfrac{1}{2}(M_1 + M_2) + \tfrac{1}{2}(M_1 - M_2)\tanh[(z - z_0)/h] \qquad (37)$$

and we took $z_0 = h/3$ so that the wing is placed below the middle plane of the shear layer.

The resulting values of lift and induced drag for elliptic wings are presented in Figs. 1-6 (graphs for several additional values of parameters are given in Ref. 9). The quantities shown are C_L/C_{L0} and $\pi A\!\!R C_{Di}/C_L^2$, where C_L and C_{Di} are the lift and induced drag coefficients in nonuniform stream based on the local velocity $U(0)$ and density $\rho(0)$, C_{L0} is the lift coefficient of the same wing at the same angle of attack α in uniform stream, and $A\!\!R$ denotes the aspect ratio of the wing. Thus

$$C_L = \frac{L}{\tfrac{1}{2}\rho(0)U^2(0)S} = \frac{\pi}{4} A\!\!R L_1 \qquad (38)$$

$$C_{Di} = \frac{D_i}{\tfrac{1}{2}\rho(0)U^2(0)S} \qquad (39)$$

$$C_{L0} = \frac{2\pi A\!\!R \alpha}{2 + \beta A\!\!R} \qquad (40)$$

where S denotes the wing area. The lift ratio C_L/C_{L0} and the induced drag factor $\pi A\!\!R C_{Di}/C_L^2$ are independent of the angle of attack, and depend on the Mach-number ratio across the stream M_0/M_1 or M_1/M_2, on the stream scale to the wing-span ratio h/s, and on the aspect-ratio parameter $\beta A\!\!R$. Their values for a uniform stream are unity. Figure 1 indicates that the variations of C_L/C_{L0} away from unity become stronger as the scale ratio h/s decreases. In the jet stream, wake stream, and nonlinear monotonic stream the changes of lift and induced drag are much stronger than in the linearly sheared stream. This is due to the direct effect of the second derivative of the Mach-number profile $M''(0)$ on the integral equation [Eq. (25)] through the logarithmic term of the kernel. The nondimensional parameter $M''(0)s^2/M(0)$ which characterizes this effect depends on the scale ratio h/s and the Mach number ratio M_0/M_1 or M_1/M_2.

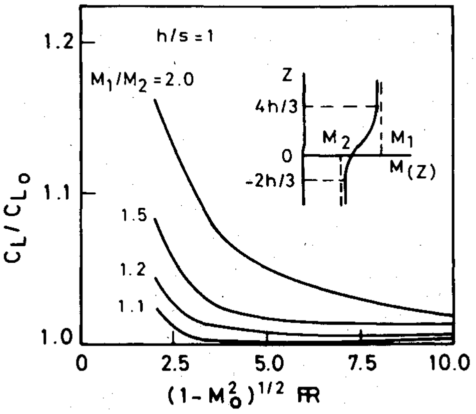

Fig. 3 Lift coefficient in nonlinear sheared stream.

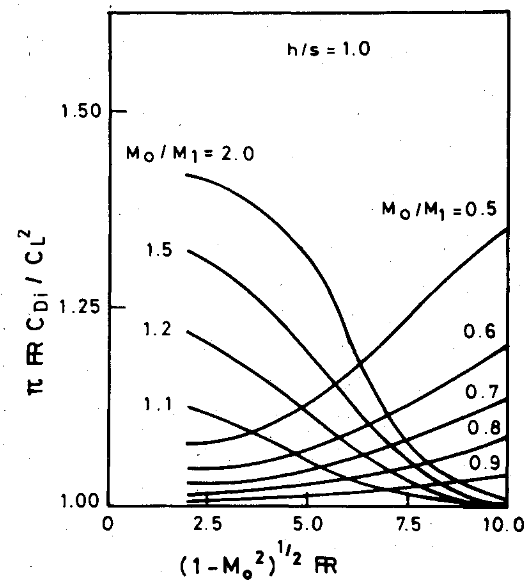

Fig. 4 Induced drag factor in jet stream and wake stream.

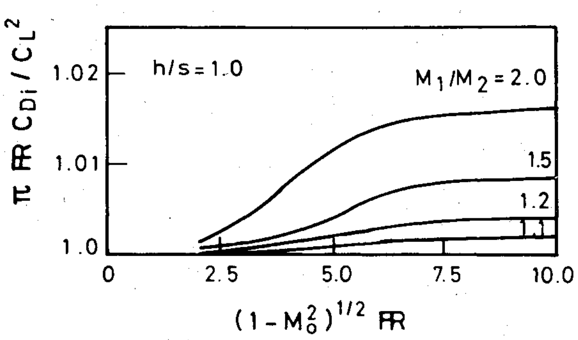

Fig. 5 Induced drag factor for linear Mach number profile.

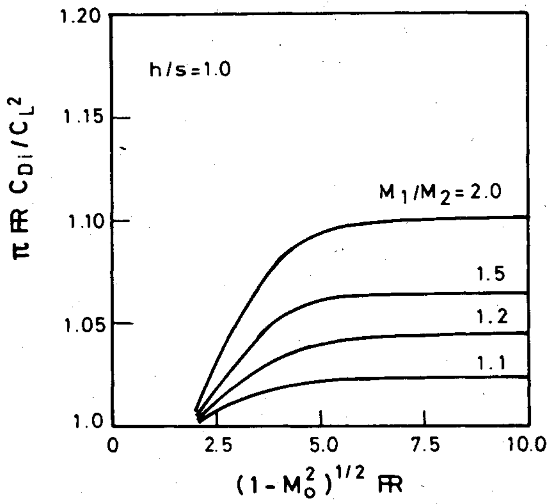

Fig. 6 Induced drag factor in nonlinear sheared stream.

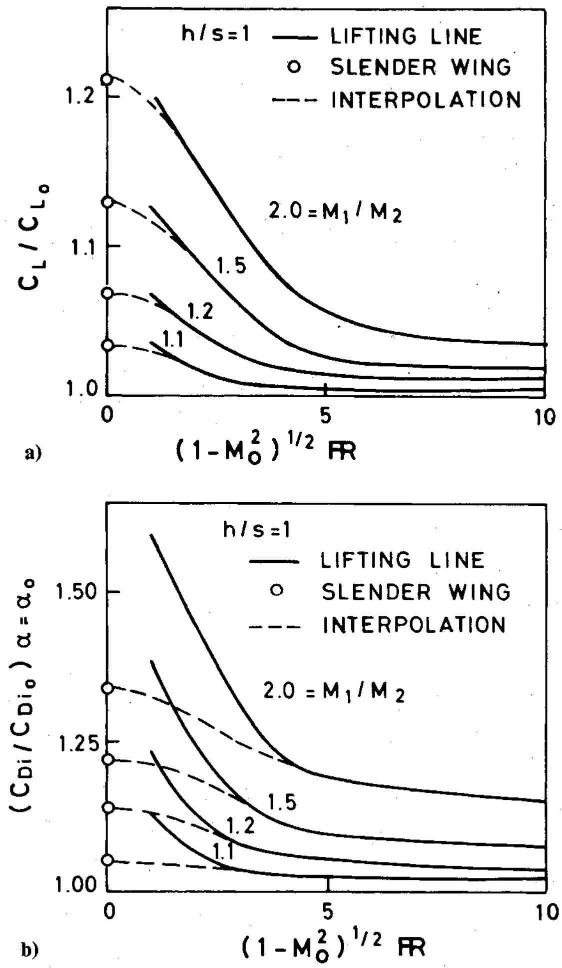

Fig. 7 Interpolation of lift and induced drag between lifting-line and slender-wing approximations: a) lift coefficient in nonlinear sheared stream, b) induced drag coefficient in nonlinear sheared stream.

For large aspect ratio ($\beta Æ \geq 8$), the lifting-line results in Figs. 1-3 are found to agree closely with the results of Weissinger[10] and Chow et al.[11] for the lift of two-dimensional airfoils in a nonuniform stream.

Figure 7 shows the values of C_L/C_{L0} and C_{Di}/C_{Di0} for low aspect ratios obtained from the slender wing theory for a nonuniform stream[6,9] together with the present results for high aspect ratios. Here $C_{Di0} = C_{L0}^2/\pi Æ$ is the induced drag coefficient in the uniform stream. It appears that the present lifting-line approximation gives reliable results for C_L/C_{L0} in the range $\beta Æ \geq 4$, and that for lower values of $\beta Æ$ an interpolation between the lifting-line and the slender wing values can be made. For the induced drag (Fig. 7b) such an interpolation does not seem feasible, which is due probably to the additional assumption $h/s = 0(1)$ or $\gg 1$ made in calculating the induced drag of high-aspect-ratio wings.

Conclusions

A lifting-surface integral equation has been obtained for wings in subsonic nonuniform parallel stream whose velocity and density vary in the vertical direction. The equation holds for general Mach-number profiles of the stream, subject to rather mild restrictions, and for wings of arbitrary planform and aspect ratio. A lifting-line integral equation for unswept or slightly swept wings with high aspect ratio in a nonuniform subsonic stream has been deduced from the lifting-surface equation by expanding the kernel asymptotically. A method has been developed for calculating the lift distribution by solving the lifting-line equation and for evaluating the pitching moment and induced drag. Numerical results have been calculated for elliptic wings with high aspect ratio in a jet stream, a wake stream, a linearly sheared stream, and a nonlinear monotonic sheared stream. The results show that the effects of a nonuniform stream on the lift and induced drag are appreciable in a wide range of the parameters. The effects become stronger with increasing ratio of maximum to minimum Mach number of the stream, decreasing ratio of the stream scale to wing span h/s, and decreasing aspect-ratio parameter $(1-M^2(0))^{1/2}\mathit{AR}$. The curvature of the Mach number profile at the wing plane is an important factor, since the parameter $M''(0)s^2/M(0)$ determines the strength of the logarithmic singularity in the integral equation for the lift distribution.

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